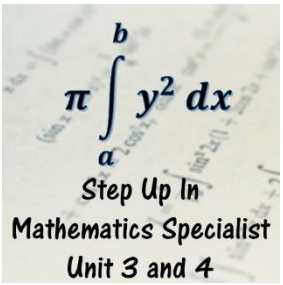
4.1 Substitution and Trig Identities

Problems Worksheet



1. Integrate the following using the suggested substitution.

   a. $\int 27(3x+2)^3\,dx$, with $u = 3x + 2$

   b. $\int 4x(x^2-1)^4\,dx$, with $u = x^2 - 1$

   c. $\int 4x(x-1)^4\,dx$, with $u = x - 1$

   d. $\int e^{\cos x}\sin x\,dx$, with $u = \cos x$

2. Integrate the following using the method of substitution, stating clearly the substitution used.

   a. $\int \frac{1}{\sqrt{9-x^2}}\,dx$

   b. $\int \frac{7}{\sqrt{3-x^2}}\,dx$

   c. $\int 22xe^{3-x^2}\,dx$

3. Determine $\int \cos^6 x\,dx$.

4. Evaluate the following definite integrals, leaving your answers in exact form. For each use the method of substitution and show your working clearly.

a. $\int_2^{-5} \frac{1}{(3-x)^2}\,dx$

b. $\int_1^2 3t^2\sqrt{2t^3-1}\,dt$

c. $\int_0^{\frac{\pi}{4}} \sin^3\theta\cos\theta\,d\theta$

d. $\int_0^{\pi^2} \frac{\sin\sqrt{x}}{3\sqrt{x}}\,dx$

5. Integrate the following using the method of substitution, stating clearly the substitution used.

   a. $\int (\tan t \sec t)\sqrt{2 - 5\sec t}\, dt$

   b. $\int e^x \sin e^x \, dx$

   c. $\int \frac{3}{x^2\sqrt{x^2+12}} dx$
   Hint: Two substitutions may be required.

6. Determine the smallest value $k \in \mathbb{R}$ which satisfies the definite integral $\int_0^k \sin^3 2x \, dx = \frac{2}{3}$.

7. Determine $\int \frac{\cos^5 x}{\sqrt{\sin x}} dx$ by clear demonstration of an appropriate substitution.